STATISTICS

- → Data (Raw Data, Frequency Distribution Table, Grouped Data)
- → Central Tendency (mean, median, mode)
- → Dispersion (Range, Mean Deviation, Standard Deviation)

Data:

Raw Data: marks (50, 45, 92, 73, 22, 73, 51)

Ungrouped Frequency Distribution Table.

|||||

Marks	No. of students.
0	2
5	7
10	15
15	12
20	3

} Frequency

Grouped Data:

→ Continuous Data (Exclusive Data)

→ Discontinuous Data (Inclusive Data)

(i) Continuous Data (Frequency)

Classes (marks)	No. of students
$\boxed{0-10}$	2
$\boxed{10-20}$	7
20-30	17
30-40	13
40-50	6

Include Exclude

(ii) Discontinuous Data:

(Exclusive Data)

Classes (Marks)	frequency (No. of students)
$\boxed{1-10}$	2
$\boxed{11-20}$	7
21 – 30	12
31 – 40	9

Include Exclude.

e.g. Convert following discontinuous Data into Continuous Data.

Classes	Freq.
$\boxed{1-10}$	2
$\boxed{11-20}$	7
21–30	12
31–40	9

Gap = 1 $\longrightarrow$ $\frac{\text{Gap}}{2} = \frac{1}{2} = 0.5$

Classes	Freq.
0.5 – 10.5	2
10.5 – 20.5	7
20.5 – 30.5	12
30.5 – 40.5	9

(Continuous)

✓

Class Mark (in grouped Data)

Classes	CM $(x_i) = \frac{U.L. + LL}{2}$	Frequency (f_i)
0 – 10	$5 = x_1$	$2 = f_1$
10 – 20	$15 = x_2$	$7 = f_2$
20 – 30	$25 = x_3$	$12 = f_3$
30 – 40	$35 = x_4$	$9 = f_4$

0 – 10 → 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

10 – 20 → 10, 11, – – – 19

Class Size.

$= (\text{Class mark}_2) - (\text{Class-Mark}_1)$

$= \text{Lower limit}_2 - \text{lower limit}_1$

$= \text{Upperlimit}_2 - \text{Upperlimit}_1$

Central Tendency (Mean, median, Mode)

200 ↙ ↘ 100 100

Mean (Average) = $\dfrac{\text{Sum of observations}}{\text{total no. of observations}}$

For Raw Data

$x_1, x_2, x_3, \ldots, x_n$ ← Values

$$\text{Mean} = \bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}}$$

For Ungrouped Frequency Distribution Table.

(Marks) Values (x_i)	(No. of students) Frequency (f_i)	$x_i f_i$
x_1	f_1	$x_1 f_1$
x_2	f_2	$x_2 f_2$
$\vdots$	$\vdots$	$\vdots$
x_n	f_n	$x_n f_n$
	$n = \sum_{i=1}^{n} f_i$	$\sum_{i=1}^{n} x_i f_i$

$\Rightarrow \text{mean} = \bar{x} = \dfrac{\sum x_i f_i}{n}$

$$\boxed{\bar{x} = \frac{\sum f_i x_i}{\sum f_i}} \star$$

Mean for Grouped Data. (Continuous or Discontinuous Data)

Classes	Class Mark (x_i) Class $= \left(\frac{UL+LL}{2}\right)$	Frequecy (f_i)	$x_i f_i$	$d_i = x_i - A$ (d = deviation)	$f_i d_i$	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
$a_1 - a_2$	x_1	f_1	$x_1 f_1$	$d_1 = x_1 - A$	$f_1 d_1$	$u_1 = \frac{x_1 - A}{h}$	$f_1 u_1$
$b_1 - b_2$	x_2	f_2	$x_2 f_2$	$d_2 = x_2 - A$	$f_2 d_2$	u_2	$f_2 u_2$
$c_1 - c_2$	$x_3 = A$	f_3	$x_3 f_3$	$d_3 = x_3 - A$	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	$f_n d_n$	u_n	$f_n u_n$
$z_1 - z_2$	x_n	f_n	$x_n f_n$	$d_n = x_n - A$			
		$N = n = \sum_{i=1}^{n} f_i$	$\sum_{i=1}^{n} x_i f_i$		$\sum f_i d_i$		$\sum f_i u_i$

Direct:

$$\boxed{\bar{x} = \frac{\sum x_i f_i}{\sum f_i}}$$

$$\boxed{\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}}$$

Assumed Mean method.

$$\boxed{\bar{x} = A + \frac{\sum f_i u_i}{\sum f_i} \times h}$$

Step Deviation method

Assumed Mean = A = (any no.) (Generally mid class mark)

h = any number = (Generally class size)

Median (middle most value)

For Raw Data

For Raw Data → Ascending order → Middle → median

For Raw Data → Descending order → middle

e.g. No. of Data = odd Data: 2, 5, 7, 8, 10 median = 7

No. of Data = Even. Data → 2, 5, 7, 8, 10, 15 median $= \frac{7+8}{2} = 7.5$

For Grouped Data. (only for Continuous Data)

Classes	Frequency	Cumulative Frequency (C) (cf)
$l_1 - u_1$	f_1	$C_1 = f_1$
$l_2 - u_2$	f_2	$C_2 = f_1 + f_2$
$l_3 - u_3$	f_3	$C_3 = f_1 + f_2 + f_3$
⋮	⋮	⋮ C
$l - u$	f	⋮
$l_n - u_n$	f_n	$C_n = \Sigma f_i$
	$N = \Sigma f_i$	

← Median class

Median class : the class whose cumulative freq. is just greater than $\frac{N}{2}$.

$$\boxed{\text{median} = l + \frac{\frac{N}{2} - C}{f} \times h}$$

l = lower limit of median class

f = freq. ________

C = c.f. of preceeding class.

h = Class size

Mode (the most frequent value)

For Raw Data.

e.g. 2, 3, 7, 2, 3, 5, 3, 3 (Mode = 3)

e.g. 2, 5, 10, 10, 10, 12, 15, 15, 15, 16 mode = 10, 15

Frequency Distribution Table.

Values	freq.
5	2
10	7
15	10
20	6

mode = 15

Mode = 15

10 Highest freq.

Modal class: the class with highest frequency.

$$\boxed{\text{Mode} = l + \frac{f_m - f'}{2f_m - f' - f''} \times h}$$

For Grouped Data: (For Continuous Data)

Classes	Frequency
$l_1 - u_1$	f_1
$l_2 - u_2$	f_2
$l_3 - u_3$	f_3
⋮	⋮
$l_n - u_n$	f_n

l = lower limit of modal class

h = class size.

f_m = the highest frequency.

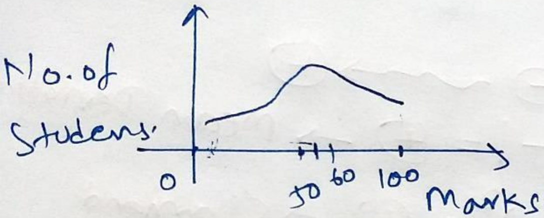
f' = freq. of preceeding class

f'' = freq. of succeeding class.

Dispersion (Degree of spreadness of Data)

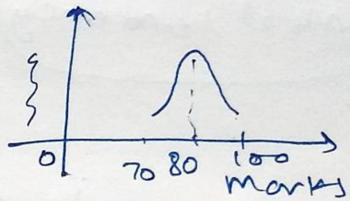
(फैलाव)

Combined class of 100 students → (A) 50 students, (B) 50 students

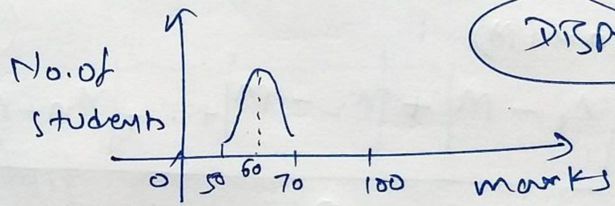


Dispersion ↑

(A) 50 students



(B) 50 students



Dispersion ↓

Measure of Dispersion

- Range = Highest Data − lowest Data
- Mean Deviation
- Standard Deviation (Variance)

e.g. Data = 2, 3, 4

Deviation about 2.5

= 2 − 2.5, 3 − 2.5, 4 − 2.5

= −0.5, 0.5, 1.5

Note: Deviation (Difference) → Always (+)ve

Deviation → (+), (−) ✓ ✓

mean = $\frac{2+3+4}{3} = 3$ ✓

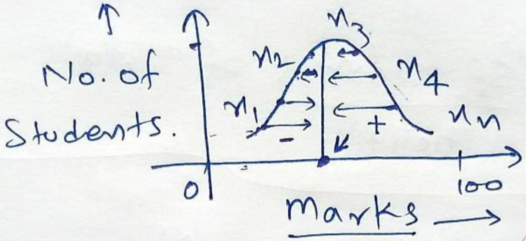
Deviation about (mean = 3) = 2−3, 3−3, 4−3

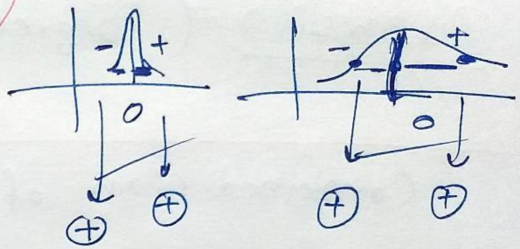
= −1, 0, 1

Sum = 0

Mean Deviation

about any central tendency





$\ominus \rightsquigarrow \oplus$

modulus $|-| = \oplus \rightarrow$ mean Deviation

Square $(-)^2 = \oplus \rightarrow$ Standard Deviation

Mean Deviation about Median(M)

$$= \frac{|x_1 - M| + |x_2 - M| + \cdots + |x_n - M|}{n}$$

☆ Raw Data

$$\text{Mean Deviation about median}(M) = \frac{\sum_{i=1}^{n} |x_i - M|}{n}$$

$$\text{mean Deviation about mean }(\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$$

Grouped Data

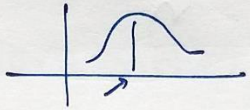
$$\text{Mean Deviation about Median }(M) = \frac{\sum f_i |\overset{\text{CM}}{x_i} - M|}{\sum f_i}$$

$$\text{Mean Deviation about mean }\bar{x} = \frac{\sum f_i |\overset{\text{CM}}{x_i} - \bar{x}|}{\sum f_i}$$

Standard Deviation. (variance) (We do square instead of MODULUS)

SD   MD

Data = $x_1, x_2, x_3, \ldots, x_n$



(Generally SD → mean के about)

Mean = $\bar{x}$

Variance (σ^2) = $\dfrac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n}$

$$\boxed{\text{Variance} = \sigma^2 = \frac{\Sigma (x_i-\bar{x})^2}{n}}$$ ← For Raw Data.

$$\boxed{\text{Standard Deviation (SD.)} = \sigma = \sqrt{\frac{\Sigma (x_i-\bar{x})^2}{n}} = \sqrt{\text{variance}}}$$

For Grouped Data.

Variance = $\sigma^2 = \dfrac{\Sigma f_i (x_i-\bar{x})^2}{\Sigma f_i}$ (CM)

Standard Deviation (SD) = $\sigma = \sqrt{\text{variance}} = \sqrt{\dfrac{\Sigma f_i (x_i-\bar{x})^2}{\Sigma f_i}}$ (CM.)

★ Mean Deviation (MD) $= \dfrac{\Sigma |x_i - M|}{n}$ या $\dfrac{\Sigma f_i |x_i - M|}{n}$

(Generally about median)

$(n = \Sigma f_i)$

★ Variance $(\sigma^2) = \dfrac{\Sigma (x_i - \bar{x})^2}{n}$ या $\dfrac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$

(Generally about mean)

$$\boxed{\sigma^2 = \text{Mean of } (x_i^2) - (\text{Mean of } x_i)^2}$$

Short cut for JEE Main

★ Standard Deviation $(\sigma) = \sqrt{\text{Variance}}$

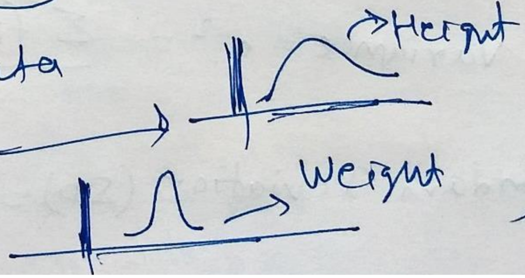
Class → 100 student.

↓

2 types Data

↳ Heights

↳ Weights



→ coefficient of variation

Coefficient of Variation (CV)

To compare the Dispersion (फैलाव) of Different Kinds of Data (Height v/s weights)

Students

Height (meters) → CV_H > CV_w ← weights (Kilograms)

~~Data~~ Spread ↑ Spread ↓

$$\boxed{CV = \frac{\sigma}{\bar{x}} \times 100}$$

σ = Standard Deviation

$\bar{x}$ = mean

$\boxed{x_1, x_2, x_3 \ldots, x_n}$

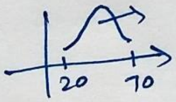
if K is added in each Data

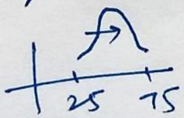
Mean → +K

median → +K

Mode → +K

MD →
σ^2 →
SD →
} → No change





if K is multiplied in each Data

Mean → ×K

Median → ×K

mode → ×K

MD → ×K

σ^2 → $\times K^2$

SD → ×K

Exercise 13.1

Q.1 Mean Deviation about Mean

4, 7, 8, 9, 10, 12, 13, 17

$$\text{Mean} = \bar{x} = \frac{4+7+8+9+10+12+13+17}{8}$$

$$= \frac{80}{8} = 10$$

$$MD = \frac{\Sigma |x_i - \bar{x}|}{n} = \frac{|4-10| + |7-10| + \cdots + |17-10|}{8}$$

$$MD(\bar{x}) = \frac{6+3+2+1+0+2+3+7}{8}$$

$$MD = \frac{24}{8} = 3$$

$$\begin{array}{r} 0.5 \\ \times 12 \\ \hline 6 \end{array}$$

Q.2 Mean Deviation about Median:

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Ascending order: $\uparrow \Rightarrow \uparrow$

10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{median} = \frac{13+14}{2} = 13.5 = M$$

(middle most value)

$$MD(M) = \frac{\Sigma |x_i - M|}{n}$$

$$= \frac{3.5 + 2.5 + 2.5 + 1.5 + 0.5 + 0.5 + 0.5 + 2.5 + 2.5 + 3.5 + 3.5 + 4.5}{12}$$

$$= \frac{28}{12} = \frac{7}{3} = 2.33$$

Q.5 Mean Deviation about Mean

x_i	f_i	$f_i x_i$	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\bar{x} = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{350}{25} = 14$$

$$MD = \frac{\Sigma f_i|x_i-\bar{x}|}{\Sigma f_i} = \frac{158}{25}$$

$$= 6.32$$

Q.7 Mean Deviation about Median

x_i	f_i	$\lvert x_i-M\rvert$	$f_i\lvert x_i-M\rvert$
5	8	2	16
7	6	0	0
9	2	2	4
10	2	3	6
12	2	5	10
15	6	8	48
	26		84

Median = the middle most value.

Here no. of values = 26

$$\text{Median} = \frac{13^{th} + 14^{th}\text{ value}}{2} = \frac{7+7}{2} = 7$$

5, 5, 5, 5, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7, 9, 9, 10, 10 ...

13^{th} 14^{th}

$$MD = \frac{\Sigma f_i|x_i-M|}{\Sigma f_i} = \frac{84}{26} = \frac{42}{13}$$

$$= 3.23$$

Q.12 Mean Deviation about Median

Age
16 – 20
21 – 25
26 – 30
31 – 35
36 – 40
41 – 45
46 – 50
51 – 55

Given

Discontinuous

Gap = 1

$\frac{\text{Gap}}{2} = \frac{1}{2} = 0.5$

LL ⊖ 0.5 UL ⊕ 0.5

$\frac{LL+UL}{2}$

Age	Number (f_i)	Class Mark (x_i)	Cumulative freq. (cf)	$\lvert x_i - M\rvert$	$f_i\lvert x_i - M\rvert$
15.5–20.5	5	18	5	20	100
20.5–25.5	6	23	11	15	90
25.5–30.5	12	28	23	10	120
30.5–35.5	14	33	37 = c	5	70
35.5–40.5	26 = f	38	63	0	0
40.5–45.5	12	43	75	5	60
45.5–50.5	16	48	91	10	160
50.5–55.5	9	53	100	15	135
	$\Sigma f_i = 100$ $N = $				735

Contin.

M = 38

Median class: 35.5 – 40.5

$$\text{median} = l + \frac{\frac{N}{2} - c}{f} \times h = 35.5 + \left(\frac{50-37}{26}\right) \times 5 = 38 = M$$

h = class size = 5

Median class: $\frac{N}{2} <$ c.f. — Just c.f.

N = 100

CM (Class Mark)

$$M.D.(M) = \frac{\Sigma f_i \lvert x_i - M\rvert}{\Sigma f_i} = \frac{735}{100} = 7.35$$

Exercise 13.2

Q.1 Mean & Variance.

6, 7, 10, 12, 13, 4, 8, 12

$\bar{x} = \text{mean} = \dfrac{6+7+10+12+13+4+8+12}{8}$

$\bar{x} = \dfrac{72}{8} = 9$

$\sigma^2 = \text{Variance} = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$

$= \dfrac{9+4+1+9+16+25+1+9}{8}$

$= \dfrac{74}{8} = 9.25$

$\sum_{r=1}^{100} r$

$\sum_{r=1}^{100} r$

Q.2 Mean & Variance of first n-natural numbers = ?

x_i — 1, 2, 3, 4, ... , n

$\text{Mean} = \bar{x} = \dfrac{1+2+3+\cdots+n}{n} = \dfrac{\Sigma x_i}{n}$

$= \dfrac{\left[\dfrac{\cancel{n}(n+1)}{2}\right]}{\cancel{n}} = \dfrac{n+1}{2}$ ✓

Variance. $= \sigma^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$

$\sigma^2 = \dfrac{\sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + (\bar{x})^2\right)}{n}$

$= \dfrac{\sum_{i=1}^{n}(x_i)^2 - 2\bar{x}\sum_{i=1}^{n}x_i + (\bar{x})^2\sum_{i=1}^{n}1}{n}$

$= \dfrac{\dfrac{\cancel{n}(n+1)(2n+1)}{6} - 2\left(\dfrac{n+1}{2}\right)\cdot\dfrac{\cancel{n}(n+1)}{2} + \left(\dfrac{n+1}{2}\right)^2\cdot\cancel{n}}{\cancel{n}}$

$$\text{Variance} = \sigma^2$$

$$= \frac{(n+1)(2n+1)}{6} - \cancel{2}\left(\frac{n+1}{\cancel{2}}\right)\left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right)^2$$

$$= \frac{(n+1)}{2}\left\{\frac{2n+1}{3} - \frac{(n+1)}{1} + \frac{n+1}{2}\right\}$$

$$= \frac{(n+1)}{2}\cdot\left\{\frac{4n+2-6n-6+3n+3}{6}\right\}$$

$$= \left(\frac{n+1}{2}\right)\cdot\left(\frac{n-1}{6}\right)$$

$$\sigma^2 = \frac{n^2-1}{12}$$

$\boxed{\text{Q.4}}$

x_i	f_i	$f_i x_i$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
6	2	12	169	338
10	4	40	81	324
14	7	98	25	175
18	12	216	1	12
24	8	192	25	200
28	4	112	81	324
30	3	90	121	363
	$n=40$	760		1736

$\bar{x}$ = 19

$$\bar{x} = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{76\cancel{0}}{4\cancel{0}} = 19$$

$$\sigma^2 = \frac{\Sigma f_i(x_i-\bar{x})^2}{\Sigma f_i} = \frac{1736}{40} = 43.5$$

Q.8

Class Mark $= \frac{LL. + UL}{2}$

Classes	x_i	f_i	$x_i f_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0-10	5	5	25	484	2420
10-20	15	8	120	144	1152
20-30	25	15	375	4	60
30-40	35	16	560	64	1024
40-50	45	6	270	324	1944
		$\Sigma f_i = 50$	$\Sigma f_i x_i = 1350$		$\Sigma f_i (x_i - \bar{x})^2 = 6600$

$\bar{x} \to 27$

$$\text{mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1350}{50} = 27$$

Class mark, mean

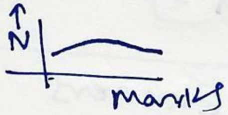
$$\text{Variance} = \sigma^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \frac{6600}{50} = 132$$

$$\text{Standard Deviation} = \sigma = \sqrt{\text{variance}} = \sqrt{132} = 11.489$$

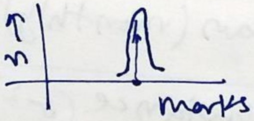
Exercise - 13.3

CV = Coefficient of Variation.

Data → more variable



Data → more stable



$CV\uparrow = \dfrac{\sigma\uparrow}{\bar{x}} \times 100$

$CV\downarrow = \dfrac{\sigma\downarrow}{\bar{x}} \times 100$

$\bar{x}$ = mean

σ = Standard Deviation $= \sqrt{\text{var.}}$

Q.1

Marks	f_A	f_B	x_i (CM)	$f_A x_i$	$f_B x_i$	$(x_i-\bar{x})^2$	$f_A(x_i-\bar{x})^2$	$f_B(x_i-\bar{x})^2$
10-20	9	10	15	135	150	876.16	7885.44	8761.6
20-30	17	20	25	425	500	384.16	6530.72	7683.2
30-40	32	30	35	1120	1050	92.16	2949.12	2764.8
40-50	33	25	45	1485	1125	0.16	5.28	4
50-60	40	43	55	2200	2365	108.16	4326.4	4650.88
60-70	10	15	65	650	975	416.16	4161.6	6242.4
70-80	9	7	75	675	525	924.16	8317.44	6469.12
	150	150		6690	6690		34179	36572

$$\bar{x}_A = \frac{\Sigma f_A x_i}{\Sigma f_A} = \frac{6690}{150} = 44.6 = \bar{x}_B$$

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

(cm)

$$\boxed{C.V. = \frac{\sigma}{\bar{x}} \times 100}$$

$$\sigma_A^2 = \frac{34179}{150} = 227.86$$

$$\sigma_A = \sqrt{227.86} = 15.09 = SD_A$$

$$\sigma_B^2 = \frac{36572}{150} = 243.813$$

$$\sigma_B = \sqrt{243.813} = 15.61 = SD_B$$

$$\sigma_A < \sigma_B$$

15.09   15.61

$$\boxed{C.V._A < C.V._B}$$

'B' is more Variable.

Q.3

	Firm A	Firm B
Earners	586	648
mean (monthly)	₹ 5252	₹ 5253
Variance (σ^2)	100	121

(i) total Amount ↑ = No. of workers ↑ × Average pay.

Same

∴ Firm 'B' has more workers

∴ Firm 'B' pays larger amount.

(ii) Greater Variability. If mean is same, then the Data with higher (σ) (B) has greater variability.

$$\boxed{C.V.\uparrow = \frac{\sigma\uparrow}{\bar{x}} \times 100}$$

Coeff. of Variation

B

Q.4

Team - A

$CV \downarrow = \frac{\sigma \downarrow}{\bar{x}} \times 100$

No. of Goals Scored (x_i)	No. of matches (f_i)	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0	1	0	4	4
1	9	9	1	9
2	7	14	0	0
3	5	15	1	5
4	3	12	4	12
	25	50	x	30

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{50}{25} = 2 = \bar{x}_A$$

Variance,

$$\sigma^2 = \frac{\Sigma f_i(x_i - \overset{\text{mean}}{\bar{x}})^2}{\Sigma f_i} = \frac{30}{25} = 1.2$$

$$SD_A = \sigma_A = \sqrt{1.2} = 1.095$$

For Team-B.

$\bar{x}_B = 2$

$\sigma_B = 1.25$

$$CV_A = \frac{\sigma_A}{\bar{x}_A} \times 100 = \frac{1.095}{2} \times 100$$

$$CV_B = \frac{\sigma_B}{\bar{x}_B} \times 100 = \frac{1.25}{2} \times 100$$

$CV_A < CV_B$

Variability of A $\downarrow$

$\Rightarrow$ Consistency of A $\uparrow$

Q.5 $\sum_{i=1}^{50} x_i = 212$ $\sum_{i=1}^{50} y_i = 261$

$\sum x_i^2 = 902.8$ $\sum y_i^2 = 1457.6$

$\boxed{\bar{x} = \frac{\sum x_i}{n}}$ $n = 50$ $\bar{y} = \frac{\sum y_i}{50}$

$$\text{Variance} = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{50} \left(x_i^2 - 2\underline{x_i}\bar{x} + (\bar{x})^2 \cdot 1\right)}{n}$$

$$= \frac{\sum x_i^2 - 2\bar{x} \cdot \sum x_i + (\bar{x})^2 \cdot \sum_{i=1}^{50} 1}{n} \to n$$

$$= \frac{\sum x_i^2}{n} - 2\bar{x} \cdot \underbrace{\frac{\sum x_i}{n}}_{\bar{x}} + \frac{(\bar{x})^2 \cdot \cancel{n}}{\cancel{n}}$$

$$= \frac{\sum x_i^2}{n} - 2(\bar{x})^2 + (\bar{x})^2$$

$$= \frac{\sum x_i^2}{n} - (\bar{x})^2 \Rightarrow \boxed{\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Var.

Variability

$\to$ S.D. $= \sqrt{\text{Var.}}$

$$\text{C.V.}\uparrow = \frac{\sigma}{\bar{x}} \times 100$$

$$\sigma_x = \sqrt{\text{Variance}_{(x)}} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\sigma_x = \sqrt{\frac{902.8}{50} - \left(\frac{212}{50}\right)^2}$$

$\boxed{\sigma_x = 0.28}$ $\boxed{\bar{x} = 4.24}$

Similarly

$$\sigma_y = \sqrt{\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2}$$

$$= \sqrt{\frac{1457.6}{50} - \left(\frac{261}{50}\right)^2} = 1.37$$

$\boxed{\bar{y} = (5.22)}$

y $\to$ weight

CV_x	CV_y
$\frac{\sigma_x}{\bar{x}} \times 100$	$\frac{\sigma_y}{\bar{y}} \times 100$
$= 6.6$	$= 26.24 \uparrow$

Miscellaneous Exercise - 13.4

Important for JEE - Main

Revision:

- Mean $= \bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n} = \dfrac{\Sigma x_i}{n}$

- Standard Deviation $= \sigma = \sqrt{\text{Variance}}$

- Variance $= \sigma^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{n}$

$$\boxed{\sigma^2 = \frac{\Sigma(x_i^2)}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} \star$$

$\to \bar{x}$

- Effect of multiplication on Values.

$\{x_1, x_2, x_3, \cdots x_n\} \longrightarrow \{Kx_1, Kx_2 \cdots Kx_n\}$

$\text{Mean}_1 = \bar{x}$	$\to \text{Mean}_2 = K\bar{x}$
$\text{Variance}_1 = \sigma^2$	$\text{Variance}_2 = K^2\sigma^2$
$SD_1 = \sigma$	$\to SD_2 = K\cdot\sigma$

Q.1 $\bar{x} = 9$, $\sigma^2 = 9.25$

$6, 7, 10, 12, 12, 13, x, y$

Mean $= \bar{x} = \dfrac{\Sigma x_i}{n} = 9$

$\Rightarrow \dfrac{6+7+10+12+12+13+x+y}{8} = 9$

$\Rightarrow 60 + x + y = 72$

$\Rightarrow x + y = 12$

$\Rightarrow \boxed{y = 12 - x}$ —— (1)

$\sigma^2 = 9.25$ (Given)

$\Rightarrow 9.25 = \dfrac{6^2+7^2+10^2+12^2+12^2+13^2 + x^2+y^2}{8} - (\bar{x})^2$

$\Rightarrow 9.25 = \dfrac{642 + x^2 + y^2}{8} - 81$

$\Rightarrow 90.25 = \frac{642 + x^2 + y^2}{8}$

$\Rightarrow 722 = 642 + x^2 + (12-x)^2$

$\Rightarrow 80 = x^2 + 144 - 24x + x^2$

$\Rightarrow 2x^2 - 24x + 64 = 0$

$\Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow \underline{x^2 - 8x} - \underline{4x + 32} = 0$

$\Rightarrow x(x-8) - 4(x-8) = 0$

$\Rightarrow (x-4)(x-8) = 0$

$x = 4$, $y = 8$ | $x = 8$, $y = 4$

$y = 12 - x$

No. = 4, 8

Q.3 6 observations $x_1, x_2, x_3 \ldots x_6$

Mean = 8

SD = σ = 4

$3x_1, 3x_2, 3x_3 \ldots 3x_6$ new observations

Mean (New) = $3 \times 8 = 24$

SD (New) = $3 \times 4 = 12$

Q.4 n-observations $\{x_1, x_2, \ldots, x_n\} \to x_i$

Mean $= \bar{x} = \frac{\Sigma x_i}{n}$

Variance $= \sigma^2 \to \frac{\Sigma (x_i - \bar{x})^2}{n}$

New (N) observations $\{ax_1, ax_2, \ldots, ax_n\} \to ax_i$

$\text{Mean}_N = \frac{ax_1 + ax_2 + \cdots + ax_n}{n}$

$\text{Mean}_N = a\frac{\Sigma x_i}{n} = a \cdot \bar{x}$

$$\text{Variance}_N = \frac{\sum (a x_i - a\bar{x})^2}{n}$$

New mean

new observations $= a x_i$
Pattern

$$\text{Variance}_N = \frac{\sum_{i=1}^{n} a^2 (x_i - \bar{x})^2}{n}$$

$$= a^2 \cdot \frac{\sum (x_i - \bar{x})^2}{n} \rightarrow \sigma^2$$

New variance $= a^2 \cdot \sigma^2$

Q.5

~~20~~

20 observations → Mean $= \bar{x} = 10$ → $\frac{\sum x_i}{20}$

→ $SD = \sigma = 2$

Var. $(\sigma^2) = 4$

{Among these 20 observations, one obs. (8) is wrong.}

(Find all information about all 19-correct observations.)

20-obs. $\longrightarrow \underbrace{x_1, x_2, \ldots, x_{19}}_{\text{Correct}}, x_{20}$

$x_{20} \leftrightarrow 8$

Mean $= 10 = \bar{x}$

$$\Rightarrow \frac{x_1 + x_2 + \cdots + x_{19} + 8}{20} = 10$$

$$\Rightarrow \boxed{x_1 + x_2 + \cdots + x_{19} = 192} \quad \text{--- (1)}$$

$\sigma^2 = \text{Variance} = 4$

$$\Rightarrow \frac{\sum (x_i^2)}{20} - \left(\frac{\sum x_i}{20}\right)^2 = 4$$

mean = 10

$$\Rightarrow \frac{x_1^2 + x_2^2 + \cdots + x_{19}^2 + 64}{20} - 100 = 4$$

$$\Rightarrow \boxed{x_1^2 + x_2^2 + \cdots + x_{19}^2 = 2016} \text{ — (2)} \quad \to \Sigma(x_i^2)$$

$$\boxed{x_1 + x_2 + \cdots + x_{19} = 192} \text{ — (1)} \quad \to \Sigma(x_i)$$

(i) wrong obs. is deleted.

No. of Obs. → 19.

$$\text{mean} = \bar{x} = \frac{\sum_{i=1}^{19} x_i}{n}$$

New

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_{19}}{19}$$

$$\bar{x} = \frac{192}{19} = 10.1$$

$$\text{Variance} = \sigma^2 = \frac{\Sigma(x_i^2)}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 \quad \Sigma(x_i^2) \to 2016$$

$$\Rightarrow \sigma^2 = \frac{2016}{19} - \left(\frac{192}{19}\right)^2$$

$$\Rightarrow \sigma^2 = 3.9889$$

$$SD = \sigma = \sqrt{3.9889} = 1.99$$

(ii) wrong obs. is replaced by '12' → x_{20}

No. of Obs. = 20 = n

obs. → $x_1, x_2, \ldots, x_{19}, 12$

n = 20

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n=20} x_i}{n}$$

$$= \frac{(x_1 + x_2 + \cdots + x_{19}) + 12}{20}$$

$$\bar{x} = \frac{192 + 12}{20} = \frac{204}{20} = 10.2$$

$$\text{Var.} = \sigma^2 = \frac{\sum_{i=1}^{20} (x_i)^2}{20} - \left(\frac{\sum_{i=1}^{20} x_i}{20}\right)^2$$

$$\sigma^2 = \frac{\boxed{x_1^2 + x_2^2 + \cdots + x_{19}^2} + 12^2}{20} - (10.2)^2$$

$$\sigma^2 = \frac{2016 + 144}{20} - (10.2)^2 = 3.96$$

$$SD = \sigma = \sqrt{3.96} = 1.98$$

(6) 50 Students.

Subject	M	P	C
$\bar{x}$	42	32	40.9
$SD = \sigma$	12	15	20

Coefficient of variation

σ = S.D.

$\bar{x}$ = Mean

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

C.V. ↑ Variability ↑

$$CV_M = \frac{\sigma_M}{\bar{x}_M} \times 100 = \frac{12}{42} \times 100 = 28.57$$

$$CV_P = \frac{15}{32} \times 100 = 46.875$$

$$CV_C = \frac{20}{40.9} \times 100 = 48.899$$

Descending 48.899 > 46.875 > 28.57 ← CV

Chemistry > Physics > Maths. Variability

↑ ↓